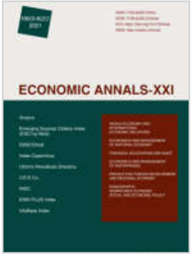


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# Rationalization of network retail management with a shift trading function based on the mathematical description of processes in the mass service area

**Abstract**

The authors highlight the high priority of rationalization in the management of the totality of transactions carried out in the sphere of network retail, in close contact with the service consumer.

For network retail objects with a shift trading function, it is important to implement a direct and permanent impact, both on the system structure and on the basic processes in the area of mass trade services. This impact focuses on studying the flow of requests, service inputs and outputs of the system, as well as the length of waiting times, and the length of queues. The success of development in such retail networks depends on the flexibility of the operations performed by the contractor in close contact with the service consumer.

It is envisaged to consider peculiarities in the rationalization of network retail management with trade turnover (flexibility) functions. The latter define the structure of the service delivery system for service consumers with processes running, in which client demands for services, as defined in the income chain, even though the intensity of the customer service flow is not constant.

The purpose of the research is to present the informative field for rationalization in network retail management with the function of shifting trade, based on the mathematical description and repeated «playing» of all processes within the area of mass service.

To represent the informative area for rationalization in network retail management with a shift trading function based on a mathematical description and repeated play of all processes within the public service area, Methods of probability theory and mathematical statistics have been used, as well as cloud computing in AnyLogic Claud environment, AnyLogic service.

The results of the study was the presentation of new possibilities for rationalizing network retail management by groups of network objects based on the concept of a mass service area, and in view of the fact that there is an $n$-channel system of mass service with an unlimited queue, where the request flow has the intensity $\lambda$, and the service flow is the intensity $\mu$.

The study was implemented with the example of one of the hubs Walmart-Salvador, uniting 90 supermarkets of the company. All Walmart hubs combine only the same supermarket type, supporting the trade changeover function within a single graph (half-yearly). Similar Walmart hubs are developed in Mexico, Great Britain, Brazil, China, Canada, South Africa, Chile, Japan, Costa Rica, Guatemala, Argentina, Honduras, Nicaragua,

El Salvador, and Ukraine. At the same time, all network nodes contain objects that apply multi-channel service systems, most common in the network retail with an unlimited queue and an option to add a new service node. It is the Walmart-Salvador hub that has a fairly high percentage of customers' refusals due to the busy service devices (this estimate ranging from 19% to 25%). As a result, Walmart's lost annual profit reaches up to USD 25.5 million approximately.
The rationalization in the management of the network retail for the Walmart-Salvador hub objects is implemented with a breakdown into 8 groups, united according to common input parameters, the latter providing a solution for the optimal number of service devices, and their required reserve and runoff are calculated, as well as efficient productivity resulting from the consistency of the input and output flows in the service channel and the stability in the mass service system. At the same time, programming for solving the problem of the management rationalization is realized using rate fixing for the basic processes in the area of mass service. In this way, mass service system sustainability is ensured, with the average timing for the application staying in the mass service system being crucial. In particular, implementing such a standard could allow avoiding losses caused by waiting for servicing and unproductive ones. Among other relevant factors are: associated timing, probability or other values (necessary for transformation operations in the characteristics of the mass service area, performed for generating target values of this indicator).
Perspectives of implementing the mathematical description of the processes in a mass-service area are in the fact that it will provide for significantly simplification in the processes of rationalizing the retail management in shift-trading facilities, regardless of the frequency of quantity and quality product range changes.
**Keywords:** Network Retail; Shift Trading; Rationalization; Management; Mass Service Area; Mass Service Model; Service User; System; Walmart Hub; Walmart-Salvador
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**Раціоналізація управління мережевим ритейлом із функцією змінної торгівлі на основі математичного опису процесів у зоні масового обслуговування**

**Анотація**
Авторами відзначено високу пріоритетність раціоналізації управління сукупністю операцій, які виконуються в сфері мережевого ритейлу, при безпосередньому контакті зі споживачем послуг. Для об'єктів мережевого ритейлу з функцією змінної торгівлі важлива реалізація прямого та постійного впливу як на структуру такої системи, так і на базові процеси в зоні масового торговельного обслуговування населення, орієнтуючись на вивчення потоків заявок на обслуговування, що надходять у систему й виходять із неї, тривалості очікування й довжини черг. Успішність розвитку таких мереж ритейлу залежить від гнучкості операцій, які виконуються виконавцем при безпосередньому контакті зі споживачем послуг.
Передбачається розглянути особливості раціоналізації управління мережевим ритейлом із функціями змінності (гнучкості) торгівлі, що визначають таку структуру системи обслуговування споживачів послуг і процесів, що в ній проходять, при якій запити клієнтів на обслуговування, що визначають ланцюг формування доходу, можуть бути оцінені, незважаючи на те, що інтенсивність потоку обслуговування покупців є величиною непостійною.
Мета дослідження – раціоналізація управління мережевим ритейлом із функцією змінної торгівлі на основі математичного опису й багаторазового «програвання» всіх процесів, які знаходяться в зоні масового обслуговування.
Для подання змістовної області раціоналізації управління мережевим ритейлом з функцією змінної торгівлі на основі математичного опису й багаторазового програвання всіх процесів, які знаходяться в зоні масового обслуговування, використано методи теорії ймовірностей і математичної статистики, а також хмарні обчислення в середовищі AnyLogic Claud, сервіс AnyLogic.
Результатом дослідження стало представлення нових можливостей раціоналізації управління мережевим ритейлом за групами об'єктів мережі на основі уявлень про зону масового обслуговування, виходячи з того, що є $n$-канальна система масового обслуговування з необмеженою чергою, в якій

потік заявок має інтенсивність $\lambda$, а потік обслуговування – інтенсивність $\mu$. Дослідження реалізовано на прикладі одного з хабів Walmart – Сальвадор, який об'єднує 90 супермаркетів компанії. Хаби Walmart об'єднують винятково однотипні супермаркети, які підтримують функцію змінності торгівлі в рамках єдиного графіка (по півріччях). Аналогічні хаби Walmart розвинені в Мексиці, Великобританії, Бразилії, Китаї, Канаді, Південно-Африканській Республіці, Чилі, Японії, Коста-Ріці, Гватемалі, Аргентині, Гондурасі, Нікарагуа, Сальвадорі, Україні. При цьому всі вузли мережі містять об'єкти, які застосовують найбільш поширені в мережевому ритейлі багатоканальні системи обслуговування з необмеженою чергою та можливістю додавання нового вузла обслуговування.
Саме хаб Walmart – Сальвадор у мережі Walmart має досить високу частку клієнтів, які відмовляються від покупки в зв'язку з зайнятістю обслуговувальних пристроїв (даний показник коливається від 19 до 25%). У результаті цього недоотриманий річний прибуток Walmart складає приблизно 25,5 млн. доларів США.
Раціоналізація управління мережевим ритейлом за об'єктами хаба Walmart – Сальвадор реалізовано в розрізі 8 груп, об'єднаних за загальними вхідними параметрами, за якими знайдено рішення щодо оптимальної кількості обслуговувальних пристроїв, встановлено їх необхідний резерв і надлишок, а також ефективну продуктивність на основі узгодженості вхідного й вихідного потоків заявок каналу обслуговування й стійкості системи масового обслуговування (СМО). При цьому програмування рішення задачі раціоналізації управління реалізовано за допомогою нормування базових процесів у зоні масового обслуговування. Це гарантує забезпечення стійкості СМО, для якого значущим є середній час перебування заявки в СМО (встановлення такої норми могло б дозволити уникнути втрат від очікування обслуговування) та супутня йому кількість часу або інші величини.
Перспективи застосування математичного опису процесів у зоні масового обслуговування полягають у можливості значного спрощення процесів раціоналізації управління ритейлом на об'єктах із функцією змінної торгівлі не залежно від частоти кількісної та якісної зміни асортименту товарів.
**Ключові слова:** мережевий ритейл; змінна торгівля; раціоналізація; менеджмент; зона масового обслуговування; модель масового обслуговування; споживач послуг; система; хаб Walmart.

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**Рационализация управления сетевым ритейлом с функцией сменной торговли на основе математического описания процессов в зоне массового обслуживания**

**Аннотация**

Авторами отмечена высокая приоритетность рационализации управления совокупностью операций, выполняемых в сфере сетевого ритейла, при непосредственном контакте с потребителем услуг. Для объектов сетевого ритейла с функцией сменной торговли важна реализация прямого и постоянного воздействия как на структуру такой системы, так и на базовые процессы в зоне массового торгового обслуживания населения, ориентируясь на изучение и обслуживание потоков заявок, поступающих в систему, и выходящих из неё, длительность ожидания и длину очередей. Успешность развития таких сетей ритейла зависит от гибкости операций, выполняемых исполнителем при непосредственном контакте с потребителем услуг.
Авторами будут рассмотрены особенности рационализации управления сетевым ритейлом с функциями сменности (гибкости) торговли, которые предполагают такую структуру системы обслуживания потребителей услуг и протекающие в ней процессы, при которых запросы клиентов на обслуживание, определяющие цепи формирования дохода, могут быть обработаны, несмотря на то, что интенсивность потока обслуживания покупателей величина не постоянная.
Цель исследования – представление содержательной области рационализация управления сетевым ритейлом с функцией сменной торговли на основе математического описания и многократного «проигрывания» всех процессов, которые находятся в зоне массового обслуживания.
Для представления содержательной области рационализация управления сетевым ритейлом с функцией сменной торговли на основе математического описания и многократного проигрывания всех процессов, которые находятся в зоне массового обслуживания, использованы методы теории вероятностей и математической статистики, а также облачные вычисления в среде AnyLogic Claud, сервис AnyLogic.
Результатом исследования стало представление новых возможностей рационализации управления сетевым ритейлом по группам объектов сети на основе представлений о зоне массового

обслуживания исходя из того, что имеется $n$-канальная система массового обслуживания с неограниченной очередью, в которой поток заявок имеет интенсивность $\lambda$, а поток обслуживания – интенсивность $\mu$.
Исследование реализовано на примере одного из хабов Walmart – Сальвадор, который объединяет 90 супермаркетов компании. Хаб – участок международной сети ритейла Walmart, о котором наличествуют все необходимые данные для математического описания процессов в зоне массового обслуживания. Хабы Walmart идеальны в качестве базы исследования, поскольку объединяют исключительно однотипные супермаркеты, поддерживающие функцию сменности торговли в рамках единого графика (по полугодиям). Аналогичные хабы Walmart развиты в Мексике, Великобритании, Бразилии, Китае, Канаде, Южно-Африканской Республике, Чили, Японии, Коста-Рике, Гватемале, Аргентине, Гондурасе, Никарагуа, Сальвадоре, Украине. При этом все узлы сети содержат объекты, которые применяют наиболее распространённые в сетевом ритейле многоканальные системы обслуживания с неограниченной очередью и возможностью на добавления нового узла обслуживания. Именно хаб Walmart – Сальвадор в сети Walmart имеет достаточно высокую долю клиентов, которые отказываются от покупки в связи с занятостью обслуживающих устройств (данный показатель колеблется от 19 до 25%). В результате недополученная годовая прибыль Walmart составляет приблизительно 25,5 млн. долларов США.
Рационализация управления сетевым ритейлом по объектам хаба Walmart – Сальвадор реализована в разрезе 8 групп, объединённых по общим входным параметрам, по которым найдено решение по оптимальному количеству обслуживающих устройств, установлен их необходимый резерв и избыток, а также эффективная производительность, продуцируемая согласованностью входного и выходного потоков заявок канала обслуживания и устойчивостью системы массового обслуживания (СМО). При этом программирование решения задачи рационализации управления реализовано с помощью нормирования базовых процессов в зоне массового обслуживания. Это гарантирует обеспечение устойчивости СМО, для которого значимо среднее время пребывания заявки в СМО (установление такой нормы могло бы позволить избежать как потерь от ожидания обслуживания, так и непроизводительных потерь) и сопутствующие ему количество времени или другие величины.
Перспективы применения математического описания процессов в зоне массового обслуживания состоят в возможности значительного упрощения процессов рационализации управления ритейлом на объектах с функцией сменной торговли независимо от частоты количественного и качественного изменения ассортимента товаров.
**Ключевые слова:** сетевой ритейл; сменная торговля; рационализация; управление; зона массового обслуживания; модель массового обслуживания; потребитель услуг; система; хаб Walmart.

## 1. Introduction

The study is aimed at rationalizing the management of retail operations totality, performed in close contact with the service consumer. It is essential to have a permanent direct impact, both on the structure of such a system and on its basic processes within the area of mass trade services. This impact is guided with the flow of service requests being input and output of the system, as well as with the waiting time duration and queues. The relevance of the study lies in the fact that network-based retailing is currently one of the fastest-growing systematic profit-making activities. The very concept of retail is limited to mass retail sales through shop trading objects with significant visitor traffic.

In the list of world's major enterprises to have sold goods worth over USD 50 billion in 2019-2020, there is the American multi-brand retailer Target (revenue of USD 75.1 billion), Tesco's largest food and non-food retailer (with revenue of USD 83 billion), French retailer network Carrefour (with revenue of USD 101 billion), American hypermarket network Home Depot Inc (with revenue of USD 108 billion), the German family company Schwarz Group (with revenue of USD 130 billion) and others (e.g., Ernst & Young Global Limited, 2020). In Ukraine, among the largest retailers (in terms of total revenue) there is a wide variety of enterprises, including ATB, Silypo, Ashan Ukraine, ECO Market, New Line, Eldorado. At the same time, the largest revenue inflows during the pandemic and the economic crisis triggered by it, have revealed retail chain enterprises oriented towards a constantly changing assortment of consumer goods, according to the demands of an end-users, household appliances, food products, building materials, and other groups of goods. For example, American Walmart, the USA's largest cash&carry chain (Costco Wholesale Corp), and Japanese retailer Aeon, supporting shifting trading practices, keep the position of the leading retailers in terms of revenue. In 2020, Costco Wholesale Corp obtained over USD 166.76 billion of revenue, Walmart had some USD 517 billion, and Aeon - USD 783 billion (according to Ernst & Young Global Limited, 2020).

This specificity is also the case in Ukraine. In this country, the ATB network, as the largest food retailer, occupies the second place among domestic revenue enterprises and supports the practice of shifting trade.

The success in development by such retail networks depends on the flexibility of the operations performed by the contractor in close contact with the service consumer. It is important to consider the rationalization in the management of the retail chain with the functions of shifts (flexible) trading defining such a structure of consumer service and processes within, where customer service demands shape the income chain, despite the level of customer service flow being not constant. This phenomenon is explained by the fact that a customer staying in a mass service area, after finding all of its channels (service devices) occupied, with a considerable duration (time) of service waiting, will commonly leave such a system without buying. This leads to risks of reduced customer flow, accumulation of dead articles in goods, disruption in the systematicity of drawing incomes, etc.

## 2. Brief Literature Review

F. Kelly and E. Yudovina (2014) as well as X. Gao and S. Deng (2016) highlight the fact that, formally, shifts trading is a part of random processes, consequently there is a need to solve probability problems related to the operation of service systems in the retail. This creates a very specific area of rationalization in network retail management, aimed at the analysis of the reasonableness, comprehension of the underlying basic processes in service systems, accepting the single actions repeated routinely as the rotation producer leading to the shift system. Such actions are described by analytic dependencies designed for a large number of tasks, though they always assume the Markov nature of the process and therefore determine only the stationary state within the system.

Masahiro Tanaka, Daichi Yanagisawa and Katsuhiro Nishinari (2019), point the intensity of the service stream fluctuating constantly with shifting trade. It is important not only to establish an analytical relationship between basic service processes, but also - their continuous monitoring statically and regulation through standardizing. This group of researchers, therefore, draws their attention to the mass service system (MSS) simulation method, widely used in retail and allowing for analysis of systems with any structure.

A range of researchers (Tšernov, 2019; Igwe, Onwumere, & Egbo, 2014, Weibing Huang, Charles-Albert Lehalle, & Mathieu Rosenbaum, 2014) use MSS simulation models, but only as a queue management tool. They indicate that in case of a too long waiting duration in queue being uncomfortable for the customer, it is possible to simulate a rise of complaints and purchase refusals. Kirill Tšernov (2019) also points out that the following factors depend on the efficiency of the mass-service simulation: the quality of trade services; the systematicity of profit making from the use of property; the scope of sales of goods, work performance, or rendering services by network retailers. The authors Weibing Huang, Charles-Albert Lehalle, and Mathieu Rosenbaum (2014) underline the decline in effectiveness due to implementation of appropriate models in service systems where the queue waiting duration is unlimited, provided that the MSS simulation tools are not a basic element for management rationalization and are not aimed at adding a new service node by users. In our opinion, the conclusion is justified. The logistics basis of the retail chain is not only represented by commercial facilities but also by the processes of selling goods to consumers in the retail space. It is only natural that we should agree with the conclusions drawn by Aiman, Norfadzlia, & Tole (2016) who indicate the necessity of activities aimed at rationalizing the usage of these facilities with an effect which cannot be fully identified through the classical system of financial performance indicators.

Alongside with the mentioned above, the studies by Karthik Ajay (2016), I. M. Toke (2014), Peter O. Peter and R. Sivasamy (2019) may be singled out, highlighting the practicability of applying mass service simulation models as a basic element of rationalizing in network retail management with a shift trading function. The authors stress the significant influence of multiple replaying the system's operation for the achievement of the target service results, provided the parameters are set up correctly. However, such researchers mostly describe possible peculiarities in advantages of service systems abstractly, without presenting their mathematical description and multiple «replaying» in a specific model.

In our opinion, the rationalization in network retail management with a shift trading function should be carried out based on a mathematical description of all processes within the service area. In this case, their environment is planned, organized, activated, and controlled based on the results of the repeated examination of the whole totality of all random events, defining their values and properties. Having completed our analysis of scientific literature, we are enabled to distinguish the following values and properties that must be a part of such a mathematical description and multiple «replaying» of the MSS functioning:

***Values:***

1) incoming flow of demands (D. Li, T. Miwa, T. Morikawa, & P. Liu, 2016; Weibing Huang, Charles-Albert Lehalle, & Mathieu Rosenbaum, 2014). The value will allow to define the law of distribution characterizing peaks of incoming buyers' claims in the system (the demands meaning the customers' addressing cashier's desk);
2) the service system structure, consisting of service nodes or service units or cashier's desks (Masahiro Tanaka, Daichi Yanagisawa, & Katsuhiro Nishinari, 2019). It allows setting up the number of service devices needed, or $n$ ;

***Additional values:***

1) an average number of channels, busy with servicing;
2) an average number of idle channels; average standby time MSS;
3) an average number of requests waiting in the queue;
4) an average number of serviced requests;
5) an average number of requests in the system (L. Chih-Chuan and W. Chin-Chieh, 2019; J. Nityangini and B. Pravin, 2017).

***Basic features:***

1) buyer service time (request) by each service node (M. Haghani and O. Ejtemai, 2014; X. Hu, S. Barnes and B. Golden, 2018);
2) Waiting discipline or set of rules governing the number of requests in the system at the same time, namely in a system with refusals, in a system with waiting or with the introduction of additional devices (M. Haghani, O. Ejtemai, and M. Sarvi, 2014; S. Chetan and J. Indira, 2020).

Additional features: load intensity; the probability of the channel being free; probability of servicing incoming requests; probability of a queue building up, probability of queue absence, probability of having to wait for being serviced (Chih-Chuan Lin, Chin-Chieh Wu, Chi-Dan Chen, and Kuan-Fu Chen, 2019; R. Obulor and B. O. Eke, 2016).

## 3. Purpose

The purpose of the study is to present the substantive area for rationalization in network retail management with the function of shifting trade, based on the mathematical description and multiple «replaying» of all processes located in the area of mass service.

## 4. Methodology and Data

A sample for implementing the study was one of the Walmart-Salvador hubs uniting 90 Walmart supermarkets in a network. This hub is a section of the Walmart international retail chain with all the necessary data available for a mathematical description of the processes in a mass service area. Walmart hubs are perfect as a research basis, as they combine only similar supermarkets, supporting the shift function in trade for fruit-and-vegetables and grocery groups, within the framework of a single graph (by half-year). Similar hubs are developed in Mexico, Great Britain, Brazil, China, Canada, South Africa, Chile, Japan, Costa Rica, Guatemala, Argentina, Honduras, Nicaragua, El Salvador. There is also a Walmart hub in Ukraine. At this, all Walmart network nodes contain objects applying the most common multichannel system in the network retail with an unlimited queue and the capability to add a new service node.

To represent the substantive area for rationalization in network retail management with a shift trading function (based on the mathematical description and multiple replays of all processes in the service area), methods of probability theory, mathematical statistics, cloud calculations in AnyLogic Claud environment were used. AnyLogic is designed to calculate all MSS indicators online.

With respect to these peculiarities, the mathematical description of all processes in AnyLogic Claud should be based on a mass-service area, assuming there is an $n$-channel unlimited-queue MSS, with the demand flow intensity $\lambda$, and the service flow with the intensity $\mu$. The rationalization in the management of the network retail is carried out at a small sample spacing (assuming the probability of the mass service system state and its performance indicators).

Thus, the Walmart may be in one of the states $S_1$, $S_2$, ..., $S_k$, ..., $S_n$, ..., numbered by the number of requests in the MSS.

The $S_0$ state indicates that there are no requests in the system (all channels are free).

The $S_1$ state indicates that 1 channel in the system is busy (but the rest are free).

The $S_2$ state indicates that two channels are busy (but the rest are free),...,$S_k$, that are busy $k$ channels (but the rest are free), and $S_n$ , that are busy all $n$ channels (but there is no queue).

The state $S_{n+r}$ state indicates that all channels are busy plus another $r$ number of requests in the queue, therefore, it is necessary to add new service devices.

The service flow intensity, as the number of requests in the Walmart MSS increases from 0 to $n$, will rise from value $\mu$ to $n\mu$, due to the increase in the number of service channels. When the number of requests in the system exceeds $n$, the intensity of the service flow remains equal to $n\mu$.

Thus, for Walmart, as for similar objects, the rationalisation in network retail management with a shift trading function is set by the initial probability data ($p$) of unlimited queue rise (or at small time spacing).

From the ratio, it follows that it is important to consider the limit probability of unlimited queue rise to be integrated into the multiple mathematical description of all processes in the Walmart Mass Area and to take into account all $S$, according to the features which we presented in Table 1.

It was predictable that the required content of a system of mathematical dependencies by $p$ (as a probability of unlimited queue rise) and $S$ shape the idea of a substantive field of rationalization in Walmart's MSS management, if it is provided with an approximate description of all value classes and properties, within the service zone. It is important to predict the MSS behavior as a real object, guided with output parameters of its efficiency, namely: nominal MSS capacity; actual MSS capacity (as a share of nominal capacity); the optimal number of service devices (channels), the optimal queue scope (up to 3 requests). The content for a system of mathematical dependencies (by $p$ and $S$), with an approximate description of all value classes and properties within the service area, is presented in Table 2.

In 2020, the Walmart-El Salvador hub facilities of the Walmart network show a fairly high percentage of customers with purchase refusals due to inavailability of service devices (this indicator ranges from 19% to 25%). These are so-called «impatient» requests. As a result, the Walmart hub had their lost annual profits at approximately USD 25.5 million over the period. The Walmart hub had lost USD 1.5 million.

To rationalize the management of the MSS for 2021, we present a mathematical description of the processes within the mass service zone in the hub, sorted by retail object groups with the function of shifting trade. The possibility of such grouping arose because the hub objects have similarity features in their input parameters (namely, the identity of the $n$-channel MSS, the request flow $\lambda$ rate, and the service flow of the MSS).

Input data for the rationalization in MSS management for the Walmart-Salvador hub facilities of the Walmart network in 2021 are presented in Table 3.

In particular, the consistent MSS behavior at the Walmart-Salvador hub objects of the Walmart network is calculated with a small step in time. With a sufficient number of iterations of the behavior of the $n$ system executed, the usual statistical processing can provide the

Table 1:
**The basis of mathematical dependencies by $p$ for taking into account the limit probability of unlimited queue rise and the initiation of the rationalization processes in network retail management with a shift trading function**

| **The model for $S_1, S_2, ..., S_k, ..., S_n$** | **Differentiation p for $S_1, S_2, ..., S_k, ..., S_n$** | | | |
|---|---|---|---|---|
| $p0$ | $p1$ | ... | $pk$ | $pn+1$ |
| $\left\|1+\frac{p}{1!*}+\frac{p^2}{2!}+\cdots+\frac{p^n}{n!}+\frac{p^{n+1}}{n!\,(n-p)}\right\|^{-1}$ | $\frac{p}{1!}*p0$ | | $\frac{p^k}{k!}*p0$ | $\frac{p^{n+1}}{n*n!}*p0$ |
| Launch management rationalization processes - launch operand (1) | 0 | 0 | 0 | 1 |

Notes: * Product of natural numbers from 1 to the actual number (including this number).

Source: Developed by the authors

Table 2:
**The algorithmic content of a system of mathematical dependencies by $p$ and $S$, with an approximate description of all classes of quantities and properties that are in the mass-service zone**

| Values and Properties | The system of mathematical dependencies | The focus area of management rationalization | Output effectiveness parameters | Output parameters |
|---|---|---|---|---|
| Load Intensity ($p$) | $\lambda * t_s$ or $\mu \rightarrow A = \lambda$, | Impact on absolute capacity $A$. The regulatory effect is achieved by rating the output flow of serviced requests, taking the rate $A = \lambda$. | Nominal $MSS$ capacity ($N = n / t_{serv}$) actual $MSS$ performance as a share of nominal capacity ($(t_{serv} / N)$*100%* *nominal perf.); the optimal number of servicing devices (channels), the optimal queue timing. | The $MSS$ sustainability ($n$ must be $> p$, then, $MSS$ is sustainable). If $p < n$ by over 20%, service process is sustainabl, but generates unproductive costs. |
| Average down time per channel in MSS $t_r$ (minutes) | $p_r * t_r$ | | | |
| The probability for the channel being free ($p0$) | $\frac{1}{1+\frac{p^1}{1!}+..+\frac{p^n}{n!}} \rightarrow p_1 = \frac{p1}{1!} * p_{0, \dots,}$ $p_n = \frac{p^n}{n!} * p_0$ | Impact on total downtime of channels and probability that each of the existing service lines is busy servicing, based on dependent algorithms to $S$: $p_1 = \frac{p1}{1!} * p_{0, \dots,} p_n = \frac{p^n}{n!} * p_0$. | | |
| Average number of channels busy servicing ($n_{av}$) | $n_{av} = \rho$ | Impact on optimal rate of busy service channels ($K = \frac{nav}{n}$), based on dependent algorithms to $S$. | | |
| Average number of idle channels ($n_{ex}$) | $n_{ex} = n - n_{av.}$ | | | |
| Probability of queuing ($p\ och$) | $\frac{p^{n+1}}{n!(n-p)} * p0$ | Impact on optimal queuing before the servicing. Hence, the probability of having to wait for being serviced is equal to the probability that all channels are busy, from dependent algorithms on $S$ and $p$. High probability of no-queue state generates unproductive costs. | | |
| Probability of queuing away, $p$ | $1 - p_{el}$ | | | |
| Probability of having to wait for being serviced ($pi$) | $\frac{p^n}{(n-1)! * (n-p)} * p0 = p\ och$ | | | |
| Average number request in queuing ($Loch$) | $\frac{n}{n-p} Poch \rightarrow \frac{Loch}{A}$ optimal value up to 3-4 request | Impact on optimal average downtime $MSS$ or $Toch = \frac{Loch}{A}$ (average waiting time per service request in queuing) based on dependent algorithms to $S$. A low value of Loch generates unproductive costs. | | |
| Average number of request service ($L_{ob}$ ) | $L_{ob} = \rho$ | Rationing of request, being serviced or waiting to be serviced, as well as the average staying time per request in the $MSS$ ($Tsmo = \frac{LCMO}{A}$), provided the number of rejected request within the hour will be $\lambda * p_1$ = 0 requests/minutes. | | |
| Average number of request in system ($LMSS$) | $Loch + L_{ob} \rightarrow$ $\rightarrow Tsmo$ | | | |

Notes:
* request flow with intensity $\lambda$, and service flow with intensity $\mu$;
** if actual MSS performance approximating 100%.

Source: Developed by the authors

desired output characteristics in efficient performance and stability of the system. Therefore, simulation mathematics is based on the repeated description for all processes within the area of mass service up to MSS reaching optimal parameters.

## 5. Results

The multiple launching of our suggested procedure for mathematical description of basic processes located in a mass service zone facilitates to highlight the specificity of the rationalization of management in the Walmart-El Salvador retail chains by object groups with the function of shifting trade. In particular, based on the results obtained (Table 4), for the network objects united into 8 groups by similarity of input parameters, the following indicators are selected:
1) the optimal number of servicing devices, required backlog for these, and excess;
2) efficient performance, produced by the consistency of input and output demand streams of service request channels and by the MSS stability.

Special attention should be paid to facilities having the following:
1) the processes in the service area are unstable without the introduction of new service devices (group 1 (half-year 2), group 2, 3, 4, 6 (half-year 1), group 7 (half-year 1, 2));
2) the service processes are stable, but $\rho$ more $n$ by over 20%. This leads to generating unproductive costs due to excess service channels performance.

According to $Loch$, such costs are evidenced at the level below two requests or a high probability no-queue state. This is typical of groups 1, 3 (half-year 1), groups 2, 6 (half-year 2), and groups 4, 5 (half-year 1, 2).

Table 3:
**Input data for the rationalization of MSS management on hub objects of Walmart-Salvador of the Walmart network, 2020**

| Objects in the Walmart network[1] | Group number | Service devices (channels), $n$, units | request flow intensity $\lambda$, requests/minute | | Absolute capacity $A$, requests/min, $A=\lambda$ | | Service flow rate | | Load intencity ($\rho=\lambda * t_{ex}$) | | Annual customer traffic, thousand | Percentage of customers purchase refusal,%[2] | Lost profits, millions, $[3] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $t_{serv}$, minutes | $\mu=1/\lambda$ | | | | | |
| half-year | | | 1 | 2 | 1 | 2 | | | 1 | 2 | | | |
| 1: 2: 16: 17: 35: 48: 52: 79 | 1 | 10 | 3 | 6 | 3 | 6 | 2 | 0.5 | 6 | 12 | 417 | 25 | 2.39 |
| 9: 13: 15: 30: 47: 56: 65: 74: 51: 81: 21 | 2 | 9 | 6 | 3 | 6 | 3 | 2 | 0.5 | 12 | 6 | 512 | 22 | 2.59 |
| 8: 19: 20: 36: 61: 76: 84: 41:55: 10: 11: 43: 33: 49: 67 | 3 | 11 | 4 | 3 | 4 | 3 | 3 | 0.33 | 12 | 9 | 617 | 19 | 2.69 |
| 12: 29: 37: 46: 77: 87 | 4 | 15 | 3 | 2 | 3 | 2 | 5 | 0.2 | 15 | 10 | 354 | 19 | 1.54 |
| 24: 25: 45: 63: 80: 86: 32: 78: 22: 58 | 5 | 22 | 4 | 3 | 4 | 3 | 4 | 0.25 | 16 | 12 | 443 | 19.8 | 2.07 |
| 4: 18: 34: 66: 70: 75: 42: 14: 26: 31: 62: 83: 89: 3: 38: 50: 27 | 6 | 16 | 5 | 4 | 5 | 4 | 3 | 0.33 | 15 | 12 | 732 | 26.8 | 4.5 |
| 23: 40: 44: 57: 71: 59: 72: 73: 85: 69 | 7 | 6 | 4 | 5 | 4 | 5 | 2 | 0.5 | 8 | 10 | 769 | 33.7 | 5.96 |
| 5: 28: 53: 54: 64: 90: 39: 60: 68: 82: 88 | 8 | 14 | 5 | 3 | 2 | 2 | 6 | 0.5 | 10 | 6 | 713 | 23.3 | 3.82 |

Notes:
[1] supermarkets by department: (San Salvador): 1-10 Ahuachapán; 11-15 Cabañas; 16-20 Chalatenango; 21-25 Sonsonate; 26-29 San Salvador; 30-33 Cuscatlán; 34-40 La Libertad; 41-50 Usulutan; 51-57 La Paz; 58-60 La Úñón; 61-66 Morazán; 67-80 San Miguel; 81-85 San Vicente; 86-90 Santa Ana;
[2] the percentage of customers with purchase refusal due to busy service devices;
[3] the indicator is defined based on the buyer's average check at USD 23.

Source: Compiled by the authors based on the following sources:
• objects of the Walmart network, $n$, $A$, $\mu$: https://corporate.walmart.com/our-story/locations/el-salvador
• annual customer traffic, Percentage of customers with purchase refusal, lost profits, $\rho$ - Data on mass service systems by objects Walmart-Salvador, 2020. *The dataset and calculations by each Group are available from the authors upon request.*

An effective solution to the rationalization in Walmart MSS management is to find a limited probability of MSS states and its performance indicators, namely, those ensuring coherence of input and output request flows and system sustainability. It should be kept in mind that the capacity indicators of the MSS influence a rapid response to the customers' requests. Result achievement is programmed by standardizing the basic processes in the Mass Service Area, which is virtually non-existent in the practice of managing network retail with a shift trading function. This is due to a lack of relevant data. At the same time, Walmart management recognizes that sustainability of the MSS is relevant to both the average time for a request staying in the MSS (setting a rate could allow to avoid both losses due to waiting for being serviced and unproductive ones), and the amount of time corresponding to it, as well as probability, or other values needed for relevant converting operations for the features of a public service area. This transformation is performed in order to generate a target value for this indicator (Ernst & Young Global Limited, 2020). This is important because the average time of a request staying in the CRM, as the basic rate in a mass service area, is considered to be the equivalent of a production rate. This is particularly important as network retailers in their actual practice often encounter so-called «impatient» requests. Such requests may leave the queue if it exceeds the optimum value of 2-3 units. The example with the Walmart-El Salvador retail network illustrates peculiarities in the use of mass service theory for setting such a basic norm based on the content of a mathematical description of processes in the represented mass service area. The specificity of using the results of a mathematical description of the processes in a mass-service area is further illustrated with the example of the Walmart-Salvador Walmart Hub Objects, grouped by the similarity of the input parameters in Group 1 (Table 5).

Table 4:
**Results for the rationalization in the MSS management within the Walmart-Salvador hub of Walmart network, based on the mathematical description of the processes in the area of mass service, 2021**

| Group | Half-year | Rationalization parameters MS | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Service stability assessment to $\rho$ ($\rho \geq n$) | Rationalization to $n$ | | | Rationalization on to *Loch*, requests | MSS capacity | | Iteration by $n$, failing to provide optimal performance output |
| | | | Optimal | Required backlog | Excess | | Nominal, requests/ minute | Actual, % of nominal capacity | |
| | 1 | 6<101 | 7 | 0 | 3 | 3.683 | 3.5 = 7 / 2 | 86%= (3 / 3.5)*100 | n=8-10 |
| 1 | 2 | 12 ≥ 102 | 14 | 4 | 0 | 2.89 | 7 | 86% | n=11-13 |
| | 1 | 12 ≥ 92 | 14 | 4 | 0 | 2.89 | 7 | 86% | n=11-13 |
| 2 | 2 | 6<9 | 7 | 0 | 2 | 3.683 | 3,5 | 86% | n=8-9 |
| | 1 | 12 ≥ 112 | 13 | 3 | 0 | 2.89 | 4.667 | 86% | n=11:12:13 |
| 3 | 2 | 9<11 | 11 | 0 | 0 | 1.937 | 3.667 | 82% | - |
| | 1 | 15 ≥ 152 | 17 | 2 | 0 | 3.93 | 3.4 | 88% | n=15,16 |
| 4 | 2 | 10<151 | 12 | 0 | 3 | 2.24 | 2.4 | 83% | n=10-11: 13-15 |
| | 1 | 16<22 | 19 | 0 | 3 | 3.93 | 4.75 | 84% | n=22-20 |
| 5 | 2 | 12<22 | 14 | 0 | 8 | 2.24 | 3.5 | 86% | n=22-15 |
| | 1 | 15≥ 16 | 17 | 1 | 0 | 3.93 | 5.667 | 88% | n=16 |
| 6 | 2 | 12<16 | 14 | 0 | 2 | 2.89 | 4.667 | 86% | n=16-15 |
| | 1 | 8 ≥ 62 | 10 | 4 | 0 | 1.63 | 5 | 80% | n=6-9 |
| 7 | 2 | 10 ≥ 62 | 12 | 6 | 0 | 2.24 | 6 | 83% | n=6-11 |
| | 1 | 10<14 | 12 | 0 | 2 | 2.24 | 6 | 83% | n=14-13 |
| 8 | 2 | 6<14 | 7 | 0 | 7 | 3.863 | 3.5 | 86% | n=14-8 |

Notes:
[1] white zone - servicing process will be stable (blue zone - $\rho < n$ for over 20-25% - servicing process stable, but still generating high unproductive costs due to excess productivity at servicing channels);
[2] gray zone - servicing process will be unstable unless new servicing devices (channels) are introduced;
[3] for the groups of network retail objects with Loch more than 3 requests, it is appropriate to complete the MSS with self-service devices.

Source: Formed by the authors in AnyLogic Claud environment based on input data for the rationalization in MSS management by hub objects of Walmart-Salvador of the Walmart net data (from Table 3) on the actual content of a system of mathematical dependencies by $p$ and $S$ with an approximate description of all value classes and properties within the mass-service zone in AnyLogic Claud for each group, namely:
- Group 1 (supermarkets 1, 2, 16, 17, 35, 48, 52);
- Group 2 (supermarkets 9, 13, 15, 30, 47, 56, 65, 74, 51, 81, 21);
- Group 3 (supermarkets 8, 19, 20, 36, 61, 76, 84, 41, 55, 10, 11, 43, 33, 49, 67);
- Group 4 (supermarkets 12, 29, 37, 46, 77, 87);
- Group 5 (supermarkets 24, 25, 45, 63, 80, 86, 32, 78, 22, 58);
- Group 6 (supermarkets 4, 18, 34, 66, 70, 75, 42, 14, 26, 31, 62, 83, 89, 3, 38, 50, 27);
- Group 7 (supermarkets 23, 40, 44, 57, 71, 59, 72, 73, 85, 69);
- Group 8 (supermarkets 5, 28, 53, 54, 64, 90, 39, 60, 68, 82, 88).

*The dataset and calculations by each Group are available from the authors upon request.*

Based on the peculiarities of a request staying in the MSS, it may be stated that rates of service flow intensity (converting a system from one state to another, based on the peculiarities of a switch trading function) are not constant values. They decrease with an increase in the number of requests in the MSS. When the reducing is no longer feasible, the number of service channels is increased or there appears a necessity to upgrade the MSS with self-service devices.

## 6. Conclusions

The rationalization in network retail management with a shift trading function is a task solved effectively based on a mathematical description and multiple replay of processes in a mass service area. For most network retail objects, this is a task that must take into account that there is an $n$-channel MSS with an unlimited queue and the possibility of adding an extra servicing channel. The request flow entering the MSS and the servicing flow (based on the number of servicing devices) have different intensity level, which may have similarities in a range of facilities. The study has shown that for hub objects Walmart-Salvador in the Walmart network there is a relatively high percentage of customers refusing to purchase due to the busy state of servicing devices (this indicator

Table 5:
**The specificity of using the results of the mathematical description of the processes in the mass-service area for standardisation by groups on the objects of Walmart-Salvador Walmart network, taking into account the function of shifting trade, 2021**

| Group | Probability of $p_{1...n}$ channels being busy servicing to identify possible states $S_1, S_2, ..., S_k, ..., S_n$, % | Indicators for conversion transactions of mass-service area features for Norm Generation T MSS | | | | | | H T MSS, minutes |
|---|---|---|---|---|---|---|---|---|
| | | *K*, % | Average idle time per channel (minutes) | | | *Toch*, minutes | *LMSS*, units | |
| | | | *poch* | *p* | *pi*[5] | | | |
| 1 | half-year 1: $p_1$= 0.00947: $p_2$= 0.0284: $p_3$= 0.0568, $p_4$= 0.0853: $p_5$= 0.102, $p_6$= 0.102: $p_7$= 0.0877<br>2 half-year: $p_1$=0,0003: $p_2$=0.000336: $p_3$= 0.00135: $p_4$=0.00404: $p_5$=0.00969: $p_6$=0.0194: $p_7$=0.0332: $p_8$=0.0498: $p_9$=0.0664: $p_{10}$=0.0797: $p_{11}$=0.087: $p_{12}$=0.087: $p_{13}$= 0.0803: $p_{14}$= 0.0688 | 90 | 0.53<br>0.4 | 0.474<br>0.587 | 0.614<br>0.48 | 1.228<br>0.48 | 9.7<br>14.89 | 3.23<br>2.43 |
| 2 | half-year 1: $p_1$= 0.056: $p_2$= 0.000336: $p_3$= 0.00135: $p_4$=0.00404: $p_5$=0.00969: $p_6$=0.0194: $p_7$=0.0332: $p_8$=0.0498: $p_9$=0.0664: $p_{10}$=0.0797: $p_{11}$=0.087: $p_{12}$= 0.087: $p_{13}$=0.0803: $p_{14}$= 0.0688<br>half-year 2: $p_1$=0.00947: $p_2$= 0.0284; $p_3$= 0.0568: $p_4$= 0.0853: $p_5$= 0.102: $p_6$= 0.102: $p_7$= 0.0877 | 90 | 0.4<br>0.53 | 0.587<br>0.474 | 0.482<br>0.614 | 0.48<br>1.23 | 14.89<br>9.7 | 2.43<br>3.23 |
| 3 | half-year 1: $p_1$=0,0056: $p_2$=0.000336: $p_3$= 0.00135; $p_4$=0.00404: $p_5$=0.00969; $p_6$=0.0194: $p_7$=0.0332: $p_8$=0.0498; $p_9$=0.0664; $p_{10}$=0.0797: $p_{11}$=0.087: $p_{12}$= 0.087; $p_{13}$= 0.0803: $p_{14}$= 0.0688<br>half-year 2: $p_1$=0.000896: $p_2$=0.00403: $p_3$=0.0121: $p_4$= 0.0272: $p_5$=0.049: $p_6$=0.0735, $p_7$=0.0945: $p_8$= 0.106; $p_9$= 0.106: $p_{10}$= 0.0957; $p_{11}$= 0.0783 | 90<br>80 | 0.413<br>0.352 | 0.587<br>0.648 | 0.482<br>0.43 | 0.72<br>0.646 | 14.89<br>10.937 | 3.72<br>3.65 |
| 4 | half-year 1: $p_1$=0,0003: $p_2$= 0,0025: $p_3$= 0.000124; $p_4$=0.000466; $p_5$= 0.0014; $p_6$= 0.00354 $p_7$= 0.00749: $p_8$= 0.014: $p_9$= 0.0234; $p_{10}$= 0.0351: $p_{11}$= 0.0479: $p_{12}$= 0.0599; $p_{13}$=0.0691; $p_{14}$=0.074: $p_{15}$= 0.074: $p_{16}$= 0.0694; $p_{17}$= 0.0612<br>half-year 2: $p_1$= 0.000359: $p_2$= 0.00179: $p_3$= 0.00598: $p_4$=0.0149: $p_5$=0.0299: $p_6$=0.0498: $p_7$= 0.0712: $p_8$ = 0.089: $p_9$=0.0989: $p_{10}$= 0.0989: $p_{11}$= 0.0899: $p_{12}$= 0.0749 | 90<br>80 | 0.46<br>0.37 | 0.541<br>0.626 | 0.52<br>0.449 | 1.3<br>1.23 | 18.9<br>12.25 | 6.3<br>6.2 |
| 5 | half-year 1: $p_1$=0,003; $p_2$=0,00255: $p_3$=0.000124: $p_4$= 0.000466: $p_5$= 0.0014; $p_6$=0.0035: $p_7$=0.00749: $p_8$= 0.014; $p_9$= 0.0234; $p_{10}$=0.0351: $p_{11}$=0.0479: $p_{12}$= 0.0599: $p_{13}$=0.0691; $p_{14}$=0.074: $p_{15}$=0.074; $p_{16}$=0.0694: $p_{17}$= 0.0612<br>half-year 2: $p_1$=0.000359: $p_2$= 0.00179: $p_3$= 0.00598: $p_4$=0.0149: $p_5$=0.0299: $p_6$=0.0498: $p_7$=0.0712: $p_8$=0.089: $p_9$=0.0989: $p_{10}$=0.0989: $p_{11}$=0.0899: $p_{12}$=0.0749 | 90<br>80 | 0.46<br>0.374 | 0.541<br>0.626 | 0.52<br>0.449 | 1.3<br>1.12 | 18.9<br>12.2 | 6.3<br>6.12 |
| 6 | half-year 1: $p_1$=0,006: $p_2$=0,0025: $p_3$=0.000124: $p_4$=0.000466: $p_5$=0.0014: $p_6$=0.0035: $p_7$=0.00749: $p_8$=0.014: $p_9$=0.0234: $p_{10}$=0.0351: $p_{11}$=0.0479: $p_{12}$=0.0599: $p_{13}$=0.0691: $p_{14}$=0.074: $p_{15}$=0.074: $p_{16}$=0.0694: $p_{17}$= 0.0612<br>half-year 2: $p_1$=0,0056: $p_2$= 0.00033: $p_3$= 0.00135: $p_4$= 0.00404: $p_5$=0.00969: $p_6$=0.0194: $p_7$=0.0332: $p_8$= 0.0498: $p_9$=0.0664: $p_{10}$= 0.0797: $p_{11}$= 0.087: $p_{12}$= 0.087: $p_{13}$= 0.0803: $p_{14}$= 0.0688 | 90 | 0.46<br>0.413 | 0.54<br>0.587 | 0.52<br>0.482 | 0.78<br>0.72 | 18.9<br>14.9 | 3.78<br>3.723 |
| 7 | half-year 1: $p_1$=0.00221: $p_2$=0.00885: $p_3$=0.0236: $p_4$= 0.0472: $p_5$=0.0755: $p_6$=0.101: $p_7$=0.115; $p_8$=0.115; $p_9$=0.102: $p_{10}$=0.0818<br>half-year 2: $p_1$=0.000359: $p_2$=0.00179: $p_3$= 0.00598: $p_4$=0.0149: $p_5$=0.0299: $p_6$=0.0498: $p_7$=0.0712: $p_8$=0.089: $p_9$=0.0989: $p_{10}$=0.0989: $p_{11}$=0.0899: $p_{12}$=0.0749 | 80 | 0.33<br>0.37 | 0.67<br>0.63 | 0.41<br>0.45 | 0.41<br>0.45 | 9.6<br>12.2 | 2.4<br>2.45 |
| 8 | half-year 1: $p_1$=0.00036: $p_2$= 0.00179: $p_3$=0.00598: $p_4$= 0.0149: $p_5$=0.0299: $p_6$=0.0498: $p_7$=0.0712: $p_8$=0.089: $p_9$=0.0989: $p_{10}$=0.0989: $p_{11}$=0.0899: $p_{12}$=0.0749<br>2 half-year: $p_1$=0.0095: $p_2$=0.0284: $p_3$=0.057: $p_4$= 0.0853: $p_5$=0.102: $p_6$= 0.102: $p_7$=0.088 | 80<br>90 | 0.37<br>0.52 | 0.63<br>0.47 | 0.44<br>0.61 | 0.45<br>1.22 | 12.2<br>9.68 | 2.45<br>3.23 |

Notes: *K* is Busy in Servicing indicator for channels; *poch* is Probability of queuing; *Toch* is Average downtime per channel in MSS; *LCMO* is Average number of requests in the system (i.e. requests already serviced through the channel and those still waiting to be serviced); *H T MSS* is standard average time for request staying in the MSS.

Source: Formed by the authors in AnyLogic Claud environment based on Table 4 data for *Loch, n*. Data on the actual content of a system of mathematical dependencies by *p* and *S* with a approximate description of all classes of values and properties that are in the mass-service zone in AnyLogic Claud for each group, namely:

- Group 1 (supermarkets 1, 2, 16, 17, 35, 48, 52);
- Group 2 (supermarkets 9, 13, 15, 30, 47, 56, 65, 74, 51, 81, 21);
- Group 3 (supermarkets 8, 19, 20, 36, 61, 76, 84, 41, 55, 10, 11, 43, 33, 49, 67);
- Group 4 (supermarkets 12, 29, 37, 46, 77, 87);
- Group 5 (supermarkets 24, 25, 45, 63, 80, 86, 32, 78, 22, 58);
- Group 6 (supermarkets 4, 18, 34, 66, 70, 75, 42, 14, 26, 31, 62, 83, 89, 3, 38, 50, 27);
- Group 7 (supermarkets 23, 40, 44, 57, 71, 59, 72, 73, 85, 69);
- Group 8 (supermarkets 5, 28, 53, 54, 64, 90, 39, 60, 68, 82, 88).

*The dataset and calculations by each Group are available from the authors upon request.*

varying within a range from 19% to 25%). As a result, Walmart's lost annual profit is approximately USD 25.5 million. Based on this specificity, there is a need to rationalize the retail management by the hub objects of Walmart-El Salvador's network. The value of lost annual profits should be minimized. The problem solution for the hub objects is implemented broken down in nine groups with common input parameters, where a solution is found for the optimal number of servicing devices, their required backup, and extra number, as well as efficient performance (generated by the consistency of the input and output request flows in a service channel and by the MSS stability).

At the same time, the programming of the solution of the problem of management rationalization is realized using basic processes standardizing in the area of mass service. This guarantees the MSS sustainability for which the important factors are: average time for request staying in the MSS (setting such a standard would allow to avoid losses due to waiting for service and unproductive ones) and the associated amount of time, as well as probability or other values required for the MSS transformation operations. The transformation is performed in order to generate target values for this indicator. For example, groups of objects at the Walmart-El Salvador hub illustrate the specificity of the basic processes standardizing in a mass-service area, taking into account the shifting trade function fro each Group as follows:

***Results for Group 1*** (supermarkets 1, 2, 16, 17, 35, 48, 52) showed that the stability provides for a nominal capacity of MSS within the limits of 3.5 requests per minute and actual capacity at 86% of the rated one. The values in the first half-year will be achieved in keeping the average time rate for request staying in the MSS up to 3.228 minutes, with three servicing channels being transferred to a temporary backup. The second half-year is marked by an increase in the number of requests in MSS, resulting in the system stability provided by an increase of MSS nominal capacity up to 7 requests per minute. The values will be achieved at updating the of the average time rate for request staying in the MSS up to 3.5 / 2.428 minutes, with the number of service channels at 14 units.

***Results for Group 2*** (supermarkets 9, 13, 15, 30, 47, 56, 65, 74, 51, 81, 21) showed the reduction of MSS nominal capacity from 7 to 3.5 requests per minute (at a steady actual capacity). For balancing input and output request flows need to be programmed at increasing the average time for request staying in the MSS up to 3.228 minutes with forming a temporary backup of 7 servicing channels.

***Results for Group 3*** (supermarkets 8, 19, 20, 36, 61, 76, 84, 41, 55, 10, 11, 43, 33, 49, 67) showed the reduction of MSS nominal capacity to 3.667 requests per minute with actual capacity 82% over the rated one. In half-year 1, it is reasonable to reach the average time of a request staying in the MSS up to 6.3 minutes, upgrading service channels by 3 units. In half-year 2, in order to eliminate the MSS excess capacity and reach balancing between input and output request flows, it is necessary to program reducing of the average time for request staying in the MSS up to 6.2 minutes and transfer two service channels to the temporary backup.

***Results for Group 4*** (supermarkets 12, 29, 37, 46, 77, 87) showed the reduction of MSS rated capacity up to 2.4 requests per minute at the actual capacity of 82% over the rated one. To ensure MSS sustainability state it is necessary to program reducing in the average time of request staying in the MSS up to 6.2 minutes, transferring three service channels to the temporary backup for half-year 2. However, taking into account the demand for service channels for half-year 2, it is necessary to upgrade the backup by two units.

***Results for Group 5*** (supermarkets 24, 25, 45, 63, 80, 86, 32, 78, 22, 58) showed a cutdown of nominal capacity of MSS from 4.75 to 3.5 requests per minute and the actual capacity 86% over the rated one. In half-year 1, it is appropriate to reach average time for a request staying in the MSS up to 6.3 minutes, with three service channels being transferred to the temporary backup. In half-year 2, MSS will remain in a sustainable state, although it is reasonable to program reducing in the average time of a request staying in the MSS up to 6.12 minutes, with transferring 5 servicing channels to the temporary backup.

***Results for Group 6*** (supermarkets 4, 18, 34, 66, 70, 75, 42, 14, 26, 31, 62, 83, 89, 3, 38, 50, 27) showed the reduction in MSS rated capacity up to 4.667 requests per minute and the actual capacity 86% over the rated one. In half-year 1, it is reasonable to reach the average timing for request staying in the MSS up to 3.78 minutes and upgrade the service channels by one unit. In half-year 2, the MSS will remain in a sustainable state, although it is appropriate to program reducing in the average time of a request staying in the MSS up to 3.723 minutes, with transferring two service channels to the temporary backup.

***Results for Group 7*** (supermarkets 23, 40, 44, 57, 71, 59, 72, 73, 85, 69) showed some reduction in MSS nominal capacity at up to 6 requests per minute and the actual capacity 83% over the rated one. In half-year 1, it is reasonable to reach the average time for a request staying in the MSS up to 2.4 minutes, by expanding the service channels stock by 4 units. In half-year 1, it is appropriate to reach the average time of a request staying in the MSS up to 2.45 minutes and expanding the service channels stock by 2 units.

***Results for Group 8*** (supermarkets 5, 28, 53, 54, 64, 90, 39, 60, 68, 82, 88) showed some reduction of MSS nominal capacity up to 3.5 requests per minute and the actual capacity 86% over the rated one. In half-year 1, the required rated average time of a request staying in the MSS must be equal to 2.45 minutes with the number of service channels to be reduced by 2 units. In half-year 2, the required rated average time of a request location in the MSS must be equal to some 3.23 minutes. It is reasonable to transfer 5 service channels to the temporary backup (to remove the excess capacity).

Most supermarket groups in the Walmart-El Salvador hub have shown the following results: a necessity in upgrading the MSS with CMO self-servicing devices in order to generate the $T\ MSS\ norm$; specificity rating by indicators required for the generation of the relevant rate.

The applicability roadmap for the presented mathematical description of the processes in a mass-service area will lie in the possibility of a significant simplification of the rationalizing processes in the retail management at shift-trading facilities, regardless of the frequency of changes in product range quantity and quality.

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